Preimage search using low communication cost parallel Grover algorithm

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Introduction

Preimage

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It is desirable that given an output it should be computationally infeasible to find any input that hashes to that output.

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Consider $n = 128$ and $H = \text{AES}$ and 0 fixed as a plain text, i.e., $H(x) = AES_x(0)$, where x is a key.

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The complexity to find one key is 2^{128} guesses.

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Brute-force search for one preimage

Let H be a function that $H : \{0,1\}^n \rightarrow \{0,1\}^n$.
The brute force is to check every input x given an output y. The time complexity will be 2^n guesses using classical computers.

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If we apply Grover's algorithm , using a quantum computer, the complexity decreases to $2^{n/2}$ guesses.

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Brute-force search for multi target preimages

Let H be a function that $H : \{0,1\}^n \to \{0,1\}^n$.
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However, we have a set of output y's, i.e., $Y = \{y_1, y_2, \ldots, y_t\}$ and we want to find one y_i.
Now, we verify every input x with set of output Y. If we **ignore several costs**, the complexity decreases to $2^n/t$ guesses in a classical computer.
If we apply Grover's algorithm, using a quantum computer, the complexity decreases to $2^{n/2}/t^{1/2}$ guesses.

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- Classical computer:
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  - Multi target: $t * 2^n/t$
- Quantum computer:
  - Single target: $2^{n/2}$
  - Multi target: $t * 2^{n/2}/t^{1/2}$

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- However, it is pre-quantum.

NIST has claimed that AES-128 is secure enough.

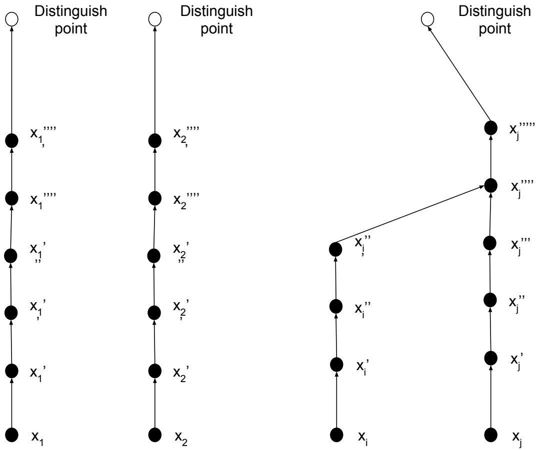
Introduction - Parallel rho method

Distinguish Point

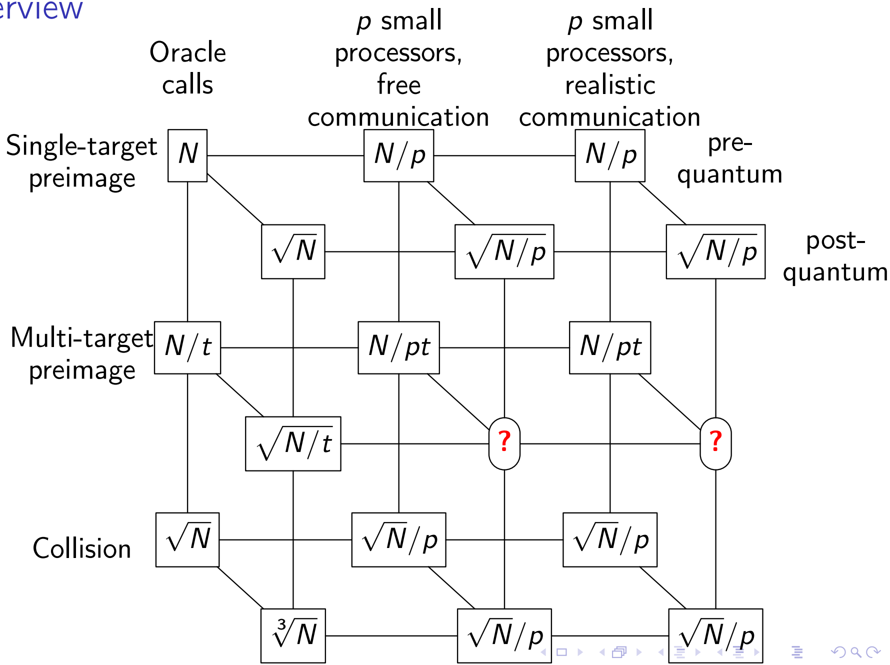
Consider $H : \{0,1\}^b \to \{0,1\}^b$
Take x an input of H, $x' = H(x)$.
Thereafter, take x' and apply H again, $x'' = H(x')$.
It is possible to do it n times (H^n), until a given condition is satified. In our case, we want the first $0 < d < b/2$ bits as 0.
$H_d^n(x)$ means d bits of x, computed n times.

Introduction - Parallel rho method

Distinguish Point



Overview



Oracle calls
p small processors, free communication
p small processors, realistic communication
pre-quantum
post-quantum
Single-target preimage
Multi-target preimage
Collision
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N/p
$\sqrt{N}$
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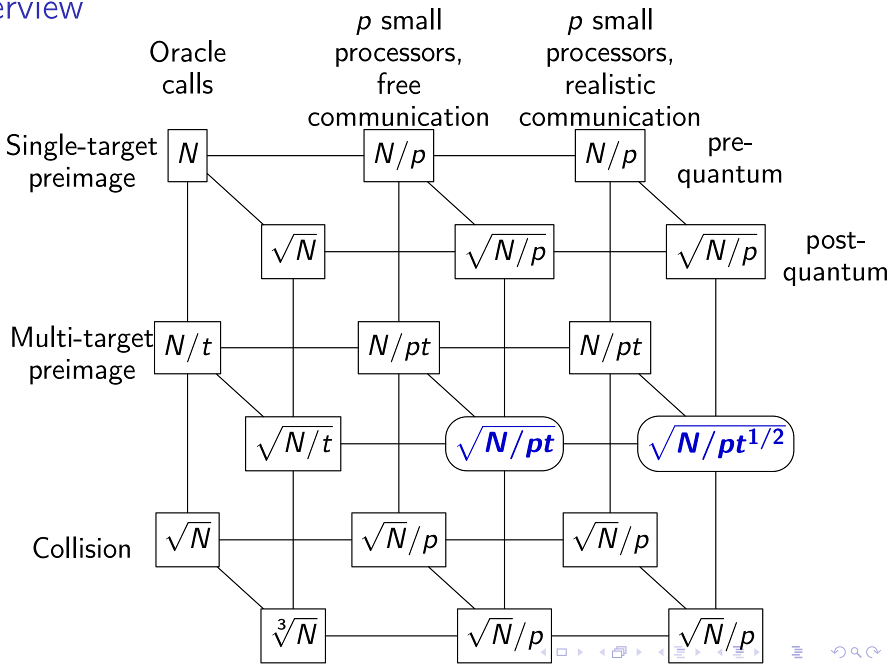

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Distinguish point in quantum setting

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- The sorting needs to be reversible too.

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Using classical computers

Example to compute $H^3(x)$:

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time 0: x y

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Example to compute $H^3(x)$:

time 0:	x	y	
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Trade-off from Bennett–Tompa

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time 0:	x	y	0	0	0

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time 4:	x	$y + H^3(x)$	$H(x)$	0	0
time 5:	x	$y + H^3(x)$	0	0	0

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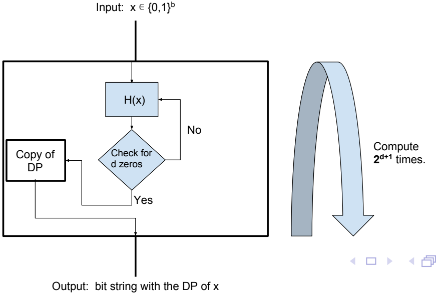

$$
\begin{array}{ccc}
x_1 & \cdots & x_t \\
\downarrow & & \downarrow \\
H_d(x_1) & \cdots & H_d(x_t) \\
\vdots & & \vdots \\
H_d^n(x_1) & \cdots & H_d^n(x_t)
\end{array}
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$$H_d^n(y_i) \stackrel{?}{=} H_d^n(x_j)$$

Reversibility

Reversibility of distinguish point

- Bennett-Tompa technique to build a reversible circuit for H^n;
- It is possible to achieve $a + O(b \log_2 n)$ ancillas and gate depth $O(gn^{1+\epsilon})$.

[3] Efficient distributed quantum computing
Beals, Robert and Brierley, Stephen and Gray, Oliver and Harrow, Aram W. and Kutin, Samuel and Linden, Noah and Shepherd, Dan and Stather, Mark

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Reversibility of sorting on a mesh network

- Using the sorting strategy from "Efficient distributed quantum computing"[3];
- We used Odd-even mergesort;
- It is possible to perform the sorting of t elements using $O(t(b + (\log t)^2))$ ancillas and $O(t^{1/2}(\log t)^2)$ steps.

[3] Efficient distributed quantum computing
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Finding t-images

Fix images $y_1, \ldots, y_t$. We build a reversible circuit that performs the following operations:

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- If there is a collision, say a collision between the chain end for x_i and the chain end for y_j: recompute the chain for x_i, checking each chain element to see whether it is a preimage for y_j.

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- Output 0 if a preimage was found, otherwise 1.

Example

- Imagine a function $H : \{0,1\}^{40} \rightarrow \{0,1\}^{40}$;

Example

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- Consider $t = 2^8$ and $p = 2^8$, for this example.

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- Consider $t = 2^8$ and $p = 2^8$, for this example.
- The probability to find one preimage is roughly $t^{5/2}/N = (2^8)^{5/2}/(2^{40}) \approx 2^{-20}$;
- Each processor is going to use $\sqrt{N/pt^{3/2}}$ iterations; $\sqrt{2^{40}/2^8((2^8)^{3/2})} = \sqrt{2^{40}/2^{20}} = 2^{10}$ iterations.
- Overall, we get $(2^8)^{1/4}$ speedup from attacking 2^8 targets.

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- Each processor is going to use $\sqrt{N/pt^{3/2}}$ iterations;
- $\sqrt{2^{128}/2^{30}(2^{30})^{3/2}} \approx \sqrt{2^{128}/2^{75}}$
- $= \sqrt{2^{53}} \approx 2^{26}$ iterations.

Conclusion & What's next?

Conclusion:

- Circuit uses $O(a + tb + t(\log t)^2)$ ancillas;
- Depth of $O(\sqrt{N/pt^{1/2}}(gt^{\epsilon/2} + (\log t)^2 \log b))$;
- Approximately $\sqrt{N/pt^{3/2}}$ iterations.
- Created the circuit using quantum simulator for AES[4] (libquantum instead of LiQ$Ui|\rangle$);

[4] Applying Grover's algorithm to AES: quantum resource estimates Grassl, Markus and Langenberg, Brandon and Roetteler, Martin and Steinwandt, Rainer

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What's next?

- Check for the real number of qubits/gates;
- Is it possible to improve?

[4] Applying Grover's algorithm to AES: quantum resource estimates Grassl, Markus and Langenberg, Brandon and Roetteler, Martin and Steinwandt, Rainer

Questions

Thank you for your attention.
Questions?

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